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ON THE MANNER OF GROWTH OF A CURRENT IN THE COIL OF A NEARLY-CLOSED ELECTROMAGNET AS INFLUENCED BY THE WIDTH OF THE AIR GAP.

By B. O. Peirce.

ON THE MANNER OF GROWTH OF A CURRENT IN THE COIL OF A NEARLY-CLOSED ELECTROMAGNET AS INFLUENCED BY THE WIDTH OF THE AIR GAP.

I

By B. O. Peirce.

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In the course of a set of experiments on the time-lag in the magnetization of iron, which are being carried on in the Jefferson Laboratory, it has been found desirable to use, in series with the testing apparatus, one or more large electromagnets to increase the inductance of the whole circuit so greatly that the effects of sudden small changes in the inductance of the testing coils may be of slight importance; and it has been necessary to study specially some of the properties of these particular magnets, since, in their cases, certain of the conditions which underlie the convenient methods usually employed in treating practical problems are not very exactly fulfilled. In order to be able to predict the behavior of a nearly-closed magnet under the conditions of the work, one needed to know the effective inductance of the magnet under given conditions, and the manner of growth of the current in the coil circuit as dependent upon the applied electromotive force, the final value of the current, and the width of the air gap. These matters, with some others, were studied, and, apropos of the extremely interesting experiments recorded in Dr. Thornton's recent article [1] on "The Magnetization of Iron in Bulk," and of the work [2] of Messrs. J. and B. Hopkinson and E. Wilson on "The Propagation of Magnetization of Iron as Affected by the Electric Currents in the Iron," I propose to give briefly in this paper the results of tests made on one of the electromagnets I have used, as illustrating some characteristics of nearly-closed, massive iron cores. With these results I wish to compare some others obtained from a large closed electromagnet with finely laminated core.

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[1] The Philosophical Magazine, 8, 1904.

[2] The Philosophical Transactions, 186, 1895. Proceedings of the London Institute of Electrical Engineers, 1895.

The general shape of the magnet in question is shown in Figure 1. The outside dimensions of the frame proper are about 101 cm. × 80 cm. × 40 cm. The base is of cast iron and of rectangular cross-section (20 cm. × 40 cm.), the cylindrical arms are of soft wrought iron 25 cm. in diameter, the rectangular pole pieces are 4.5 cm. thick, and the area of each of the opposed faces is about 580 square centimeters. The four coils, which were commonly used in series, have together 2823 turns and a resistance at 20° C. of about 12.4 ohms; the magnet weighs about 1500 kilograms.

The electromagnetic induction within so large a solid core as that of this magnet practically attains its final value, as is well known, only after an appreciable length of time. This time, for a given value of the electromotive force in the coil circuit, depends upon the amount

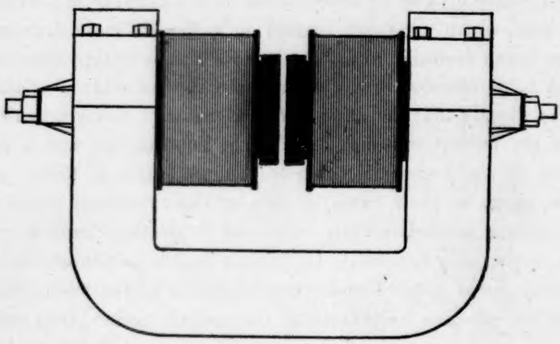

FIGURE 1.

of non-inductive resistance in the circuit outside the magnet, and, for a given value of the final exciting current in the coil, depends upon the applied electromotive force; under favorable circumstances it may be 200 seconds. At the outset Mr. J. Coulson and I obtained a large number of hysteresis curves for the core under given conditions, but for steady currents in the coil. The curves in Figure 2 are four of a series for different widths of the air gap, each obtained after the magnet had been put a number of times in succession through a Ewing's cycle with 6 amperes as the maximum current. The ordinates show the flux density at the centre of the gap in thousands of units, and the abscissas the current in amperes for the positive descending quarter of the cycle; the rest of the figures are omitted to avoid confusion. The flux was measured by pulling a small thin coil of known dimensions, attached to a calibrated

ballistic galvanometer, out of a pocket at the middle of the gap. The cylindrical arms of the magnet were held firmly by massive yokes outside the frame, but it was necessary to insert strips of non-magnetic material in the gap to prevent it from gradually closing by the bending of the frame when strong currents were used. These "chocks" were usually so inserted, by aid of gauges made for the purpose, that just one half of the gap — divided by a vertical line from the other half — was free. Counting from the top of the diagram, the full curves correspond to gap-widths of 1.6 mm., 6.6 mm., 9.8 mm., and 19.7 mm. respectively; between the first two others are bits of two curves belonging to gap-widths of 3.2 mm. and 4.7 mm.

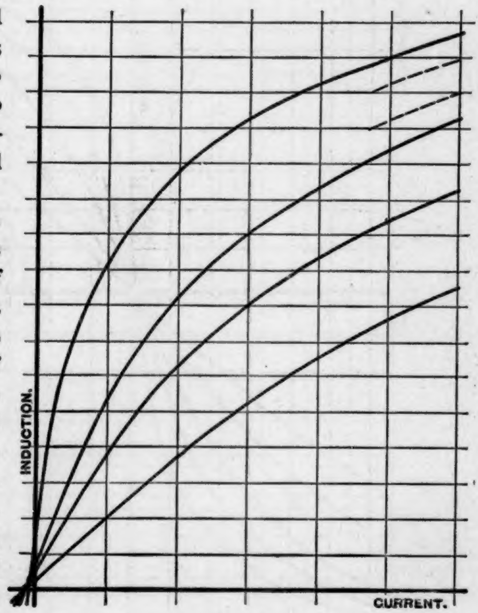

FIGURE 2.

The length of the line of induction which goes through the centre of the pole pieces is about 250 cm., and it would be easy to find the form of a curve, similar to those of Figure 2, for a closed gap by shearing[3] the upper curve of the diagram in the usual manner. When the gap was closed, a number of turns of insulated wire were wound directly about the core, and the ends of this coil were connected through an oscillograph which made its records on the same piece of paper which recorded the indications of another oscillograph in the main circuit. The main circuit contained also a massive rheostat of 200 ohms, total resistance, and the current

[3] J. Hopkinson, The Philosophical Transactions, 1885; J. and E. Hopkinson, The Philosophical Transactions, 1886; E. Hopkinson, Report of the Brit. Assoc. Adv. Sci. 1887; Ewing, Magnetic Induction in Iron and other Metals, ch. x; Du Bois, Philosophical Magazine, 1890.

in the circuit was made to grow from zero by steps so far apart in time that, in every interval, the current in the secondary circuit had time to die sensibly out. After a maximum current of the desired intensity had been attained, the current was then reduced by steps to zero, and was then built up in the opposite direction to the same maximum

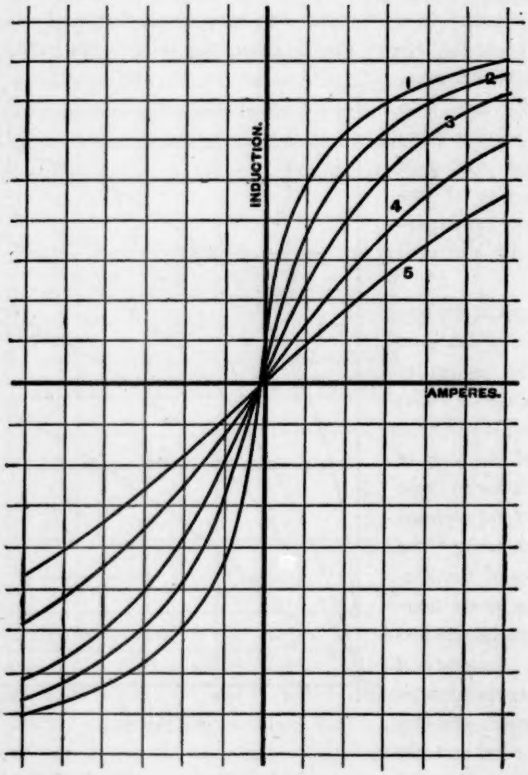

FIGURE 3.

value by steps. The areas under the oscillograph curves of the secondary circuit evidently furnish means of obtaining a hysteresis curve for the core of the magnet, but this well-known method of procedure is in my hands and for this particular magnet not quite so satisfactory as the one indicated above.

The leakage in a magnet of the form of this one is of course considerable,

even when the gap is closed, and, for a given width of gap, the ratio of the induction through a circumference of, say, 48 cm. diameter in the plane of the gap with its centre (O) at the gap-centre, to the flux density at O, depends slightly upon the intensity of magnetization of the iron. With

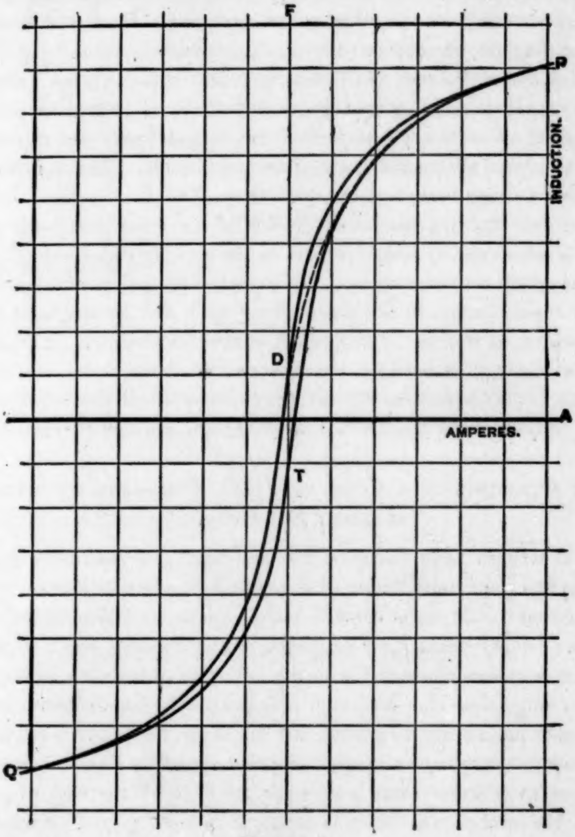

FIGURE 4.

a gap-width of 28.3 mm. this ratio increased by about 3 per cent as the strength of the current in the coils rose from a small value to 6 amperes, and was sensibly the same for ascending and descending branches of a hysteresis cycle.

The ordinates of each of the curves in Figure 3 show, in millions of

units, the mean total flux of induction, through each turn of the coils of the magnet, for a given gap-width, corresponding to currents represented in amperes by the abscissas. These curves are specimens of a set obtained experimentally: only half of each cycle — representing about 20 determined points, every one of which lies sensibly on the curve — is given lest the complete diagrams prove confusing. Reckoned from the top of the diagram, the curves correspond, respectively, to the gap-widths 1.6 mm., 4.7 mm., 9.7 mm., 19.7 mm., and 28.4 mm. Figure 4 shows a similar cycle corresponding to a gap-width of about 1.6 mm. for a maximum current of about 6.2 amperes: the magnet was put repeatedly through the cycle before the observations were made. The dotted curve DP shows a rising branch of the cycle from D to P.

It is evident that the manner of growth of a current in the coil of the magnet is influenced by eddy currents in the core, by the residual effects of past magnetic experiences and the corresponding form of a hysteresis cycle for rapid changes of the magnetizing field, and by magnetic lag, if such there be, as well as by the causes enumerated above. It is always difficult to distinguish between the effects of all these causes, and it will be well to get such help as we can from a theoretical discussion of the effect of eddy currents alone in a core of definite constant permeability.

Eddy Currents in a Core of Fixed Permeability within a Long Solenoid.

Several writers have discussed the application of Maxwell's general equations[4] to the determination of the growth of currents in coils of wire which surround solid metal cores of various forms, and Heaviside printed more than twenty years ago[5] an extremely interesting series of fifteen papers on problems connected with the induction of currents in the solid core of a long solenoid. Although it is practically impossible to subject to accurate computation the growth and the decay, under given conditions, of currents in the coils of a magnet like that shown in Figure 1, it will be instructive to consider some analogous problems in the case of a long solenoid, the solid core of which is supposed to have a fixed permeability, and to be of the same diameter as that of the iron cylinders within the coils of the magnet in question. To facilitate comparison between the numerical results of this paper and those obtained in similar cases by Heaviside, it will be convenient to use his notation, at least in part.

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[4] Treatise on Electricity and Magnetism, **2**, ch. ix.

[5] The Electrician, 1884–85; Electrical Papers, **1**, 353–416.

A long, solid, circular, iron cylinder, of specific resistance ρ and of radius a, is closely surrounded by a uniformly wound coil of wire which has N turns per centimeter of the length of the core; the outer radius of the coil is $(a + b)$ and the axis of the core is the z axis. A current (C) in the coil is accompanied by a magnetic field (H) in the core which has the direction of the z axis, and any change in the intensity of C induces in the core temporary currents, the lines of which are circles parallel to the xy plane with centres on the z axis. At any instant the value of H and that of q, the vector which gives the density of the current at any point in the core, are functions of the distance (r) from the z axis alone, and are independent of z; hence Maxwell's current equation,

$$4\pi q = \text{Curl } H, \tag{1}$$

reduces to the simple form

$$4\pi q = -\frac{\partial H}{\partial r}. \tag{2}$$

The currents in the core do not affect the intensity (H_a) of the magnetic field at the boundary of the coil, so that at every instant

$$H_a = 4\pi N C. \tag{3}$$

Since in columnar co-ordinates

$$\nabla^2 (V) \equiv \frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r \cdot \frac{\partial V}{\partial r}\right) + \frac{1}{r^2} \cdot \frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2}, \tag{4}$$

where V is any scalar function, the general equation,

$$\frac{4\pi\mu}{\rho} \cdot \frac{\partial H}{\partial t} = \nabla^2 (H), \tag{5}$$

becomes
$$\frac{4\pi\mu}{\rho} \cdot \frac{\partial H}{\partial t} = \frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r \cdot \frac{\partial H}{\partial r}\right), \tag{6}$$

where μ is the permeability of the iron, supposed constant.

When there are no currents in the core, the intensity (H) of the magnetic field in the core has at every point the boundary value (H_a), but when positively directed eddy currents exist, the intensity of the field is greater near the axis and sinks gradually, as r increases, to H_a at the boundary. If, then, L is the inductance of the coil per centimeter

of the length of the core, when the core is without currents and there are no other circuits in the neighborhood so that the magnetic field within the coil is uniform, this inductance will be increased by an amount L' when, in consequence of Foucault currents running right-handedly around the axis, the intensity of the field within the core is raised above $4\pi N C$. The contribution L' comes from a field every line of which threads every turn of the coil. We have, therefore,

$$L' = N\int_0^a \mu\,(H - H_a)\,2\pi r\,dr = 2\pi\mu N\int_0^a H r\,dr - 4\pi^2 a^2 N^2 \mu\, C, \tag{7}$$

and if the coefficient of C in the last term be denoted by L_1, the whole induction flux through the turns of the coil per centimeter of the length of the solenoid is

$$p \equiv (L - L_1)\,C + 2\pi\mu N\int_0^a H r\,.\,dr; \tag{8}$$

if w is the uniform resistance of the coil per centimeter of the length of the core and E the impressed electromotive force in the coil circuit, reckoned in the same way,

$$E - \frac{dp}{dt} = w\,C, \tag{9}$$

or
$$E = wC + (L - L_1)\frac{dC}{dt} + 2\pi\mu N\int_0^a \frac{\partial H}{\partial t}\,.\,r\,.\,dr; \tag{10}$$

or, by virtue of (6),

$$E = wC + (L - L_1)\frac{dC}{dt} + \tfrac{1}{2} N\rho a \left(\frac{\partial H}{\partial r}\right)_{r=a}. \tag{11}$$

To fix one's ideas, one might imagine every centimeter of the length of the coil (measured parallel to the core axis) to be a separate circuit containing an applied electromotive force of E absolute units (the same for every such circuit) and having a total resistance w made up of the resistance (w') of the wire actually wound on the core, and the resistance (w'') of the external part of the circuit which is non-inductive.

If, in order to determine a set of normal special solutions of the linear equation (6), we assume H to be the product of a function (T) of t alone, and a function (R) of r alone, and substitute this product in the equation, we arrive at the well-known normal form

$$e^{-a^2 t}\,[A \cdot J_0(nr) + B \cdot K_0(nr)], \tag{12}$$

where either a or n may be assumed at pleasure and the other computed by means of the equation

$$\rho n^2 = 4 \pi \mu a^2. \tag{13}$$

Since the core of the solenoid is solid, Bessel's Functions of the second kind will not be needed in the problems of this paper, and we may assume that H is expressible in an infinite series of terms of the form

$$A \cdot e^{-a^2 t} \cdot J_0 (n r). \tag{14}$$

If, after the current in the coil has been for some time steady and the core has become uniformly magnetized, the coil circuit be suddenly broken so that the duration of the spark is less than a thousandth of a second, the intensity of the magnetic field at the boundary of the core where $r = a$ falls suddenly to, and remains thereafter at, zero, and the normal form (14) will satisfy the condition $H_a = 0$, if such a value be chosen for n as shall make $n a$ a root of the equation

$$J_0 (x) = 0. \tag{15}$$

A sufficient number of these roots for the purposes of this paper can be found[6] in almost any book on Bessel's Functions, with the corresponding values of $J_1 (x)$: the first twelve are given in Table I.

The pth root in order of magnitude of the equation $J_1(x) = 0$ is denoted by x_p.

TABLE I.

p	x_p	$J_1(x_p)$	p	x_p	$J_1(x_p)$
1	2.404826	+0.519148	7	21.211637	+0.173266
2	5.520078	−0.340265	8	24.352472	−0.161702
3	8.653728	+0.271452	9	27.493479	+0.152181
4	11.791534	−0.232460	10	30.634606	−0.144166
5	14.930918	+0.206546	11	33.775820	+0.137297
6	18.071064	−0.187729	12	36.917098	−0.131325

If the intensity of the uniform magnetic field in the core before the break was H_0, we have[7] at any time (t) after the break and at any distance (r) from the axis of the core,

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[6] Byerly, Treatise on Fourier's Series and Spherical, Cylindrical, and Ellipsoidal Harmonics, p. 286; Gray and Mathews, Treatise on Bessel's Functions, p. 244; Peirce and Willson, The first 65 roots of the Equation $J_0(x) = 0$, The Bulletin of the American Mathematical Society, 1897.

[7] Byerly, Treatise on Fourier's Series, etc., p. 229; Heaviside, Electrical Papers, 1, 391.

$$H = \frac{2H_0}{a} \sum_p \frac{J_0(n_p r)}{n_p \cdot J_1(n_p a)} e^{-a_p^2 t}, \tag{16}$$

and, since $$\frac{d J_0(n r)}{d r} = -n \cdot J_1(n r),$$

$$q = \frac{+H_0}{2\pi a} \sum_p \frac{J_1(n_p r)}{J_1(n_p a)} e^{-a_p^2 t}. \tag{17}$$

In the case here treated the diameter of the core is 25 centimeters ($a = 12.5$), and for the kind of iron used, at room temperatures, we may write

$$a^2 = \frac{5.12\,(n\,a)^2}{\mu}. \tag{18}$$

The negative of the time rate of change of the total flux of induction through a cylindrical surface of radius r coaxial with the core and lying within it, is at every instant proportional to the expression for q given above. The values of $J_0(x)$ and $J_1(x)$ for every hundredth of a unit between $x = 0$ and $x = 15.50$ are given in Meissel's Tables[8] to twelve decimal places, and after the proper value of μ has been introduced into (18) and the value for r chosen, it is not very difficult, except in the case of small values of t, to compute the value of the series (S) in (17) for different epochs. If, for example, μ is 40, and if we consider a point at the boundary of the core, S has the values given in the next table. The time is of course measured in seconds.

TABLE II.

t	S	t	S
0.25	1.3120	3.00	0.1085
0.50	0.8413	4.00	0.0518
0.75	0.6285	5.00	0.0247
1.00	0.4974	6.00	0.0118
2.00	0.2227	8.00	0.0026

Figure 5 shows four curves in which S is plotted against t for $r = a$ and $\mu = 20$, 40, 80, and 160 respectively. If the circuit of a few turns of fine insulated wire wound directly on the core were closed through an oscillograph, and if μ were independent of H and there were no time lag in its magnetization, the records should show curves like these. For large

[8] Meissel, Tafel der Bessel'chen Functionen, Berliner Abhandlungen, 1888; Gray and Mathews, Treatise on Bessel's Functions, pp. 247–266.

values of μ the current would decay very slowly. The actual values of the ordinates of any curve would depend of course upon the number of turns in the secondary, the resistance of its circuit, and the magnetic constants of the solenoid. If for a fixed point in the core a curve be drawn, by plotting q against t, for each of a number of different values of μ, the ordinates of all these curves will have equal values at points where t/μ has the same value. The maximum value of q, if it has one, is independent of μ.

At a point distant one tenth of the radius of the core from the axis

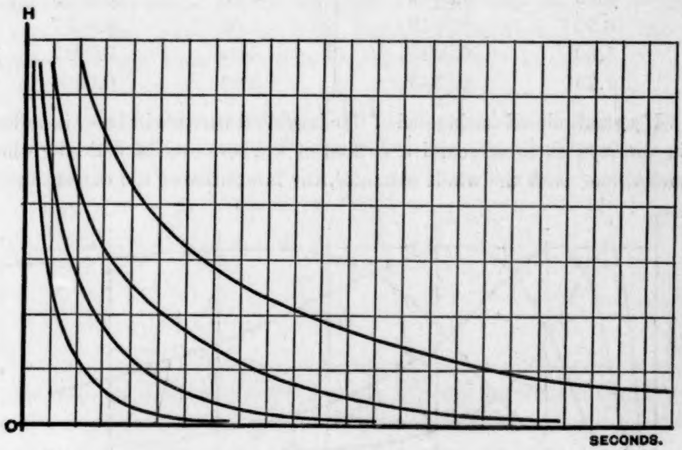

FIGURE 5.

$$q = \frac{H_0}{2\pi a} \sum \frac{J_1(\frac{1}{10} n_p a)}{J_1(n_p a)} e^{-a_p^2 t} = \frac{H_0 S'}{2\pi a}, \qquad (19)$$

and ignoring algebraic signs, we have

TABLE III.

p	$\log \frac{J_1(\frac{1}{10} n_p a)}{J_1 n_p a}$	p	$\log \frac{J_1(\frac{1}{10} n_p a)}{J_1 n_p a}$
1	9.3617	7	0.5141
2	9.8924	8	0.5009
3	0.1611	9	0.4473
4	0.3264	10	0.3395
5	0.4308	11	0.1378
6	0.4911	12	9.6398

Table IV gives to four decimal places the values of S', for $\mu = 40$ and $r = 1.25$, at a number of different epochs after the coil circuit has been broken.

TABLE IV.

t	S'	t	S'
0.17	0.0004	1.50	0.0736
0.25	0.0062	2.00	0.0520
0.50	0.0597	2.50	0.0361
0.60	0.0770	3.00	0.0250
0.75	0.0912	4.00	0.0119
0.90	0.0948	5.00	0.0057
1.00	0.0940	6.00	0.0027
1.25	0.0853	8.00	0.0008

If a small closed testing coil of fine insulated wire could be so imbedded in the iron as to surround a portion of the core coaxial with the whole and of one tenth the whole diameter, the intensities of the current in the

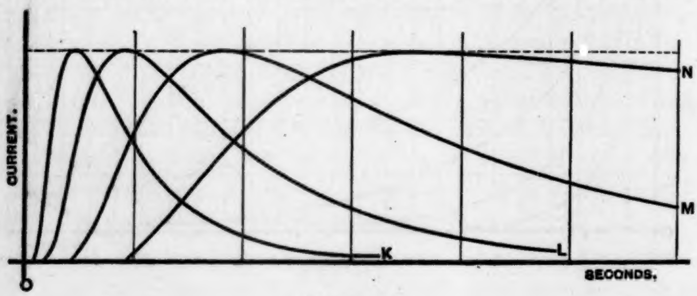

FIGURE 6.

coil at different times should be proportional to S'. Figure 6 shows four curves[9] in which S' is plotted against t for $\mu = 20$, 40, 80, and 160.

The maximum value of q at $r = a/10$ may be found by equating to zero $\partial q / \partial t$ obtained from (19)

$$\frac{\partial q}{\partial t} = -\frac{2.56\, H_0}{\pi \mu a} \sum \frac{(n_p a)^2 J_1 (\tfrac{1}{10} n_p a) \cdot e^{-c_p^2 t}}{J(n_p a)} \qquad (20)$$

If for $r = a/10$, q be plotted against t for a number of values of μ, t/μ will have the same value at the highest point of all the curves.

[9] Each curve practically coincides with the axis of abscissas for a time, and then suddenly bends sharply away from the axis. It is not easy to indicate these sharp bends in a small figure.

In order to determine the manner of growth of the current in the coil of the solenoid when the circuit is suddenly closed, it will be well to follow the usual procedure in treating analogous problems in heat conduction, and inquire first how the coil current would decay if, after the core has been uniformly magnetized by a steady coil current, the electromotive force were suddenly cut out of the coil without opening the circuit. The solution of this problem furnishes immediately the solution of the one first stated. Heaviside[10] has treated by this method a solenoid with iron core 2 centimeters in diameter. If the coil is fairly thin, so that substantially the whole flux of induction which threads its turns is the flux in the core itself, we need not distinguish between L and L_1 in equation (11) and since there is no electromotive force in the coil

$$C + \frac{N \rho a}{2 w} \left[\frac{\partial H}{\partial r}\right]_{r=a} = 0, \tag{21}$$

and by virtue of (3)

$$H_a + \frac{4 \pi N^2 \rho a}{2 w} \left[\frac{\partial H}{\partial r}\right]_{r=a} = 0, \tag{22}$$

or

$$H_a + s \cdot \left[\frac{\partial H}{\partial r}\right]_{r=a} = 0. \tag{23}$$

The constant s is to be determined from the constants of the solenoid and of the core, and we shall find it instructive to study the effect of a change in s in a problem otherwise given. We will assume at first that N, the number of turns of wire in the coil per centimeter of the length of the core, and w, the resistance in absolute units of the coil per centimeter of its length, are such as to make s unity; in the second case the value of s shall be 2.5. The first is somewhat less, the second much greater than the value which would most closely correspond to the magnet shown in Figure 1. The field intensity, H, in the core must satisfy equations (6) and (23) at every instant, and, when $t = 0$, must be equal to H_0 for all values of r.

The special solution,

$$A e^{-\alpha^2 t} \cdot J_0 (n r), \tag{24}$$

of (6), in which $$\alpha^2 = \frac{\rho (n a)^2}{4 \pi \mu a^2},$$

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[10] Heaviside, Electrical Papers, 1, 394.

satisfies (23) provided that $n\,a$ is a root of the equation

$$J_0\,(n\,a) = \frac{n\,a\,s}{a}\,J_1\,(n\,a), \tag{25}$$

and if we use the successive values [11] of n

$$1 = \sum \frac{2 \cdot J_0\,(n_p\,r)}{n_p\,a\,(1 + s^2\,n_p^{\,2})\,J_1\,(n_p\,a)}, \tag{26}$$

so that

$$H = \frac{2\,H_0}{a} \sum \frac{e^{-a_p^2 t} \cdot J_0\,(n_p\,r)}{n_p\,(1 + s^2\,n_p^{\,2}) J_1\,(n_p\,a)}, \tag{27}$$

and

$$H_a = \frac{2\,H_0\,s}{a} \sum \frac{e^{-a_p^2 t}}{1 + s^2\,n_p^{\,2}}. \tag{28}$$

In the case here treated $n\,a$ is a root of the equation

$$J_0\,(z) = \frac{2\,z}{25}\,J_1\,(z), \tag{29}$$

and it is not difficult to prove by the aid of Meissel's Tables that the first five roots have approximately the values given below.

TABLE V.

$n_1\,a = 2.2218$
$n_2\,a = 5.1171$
$n_3\,a = 8.0624$
$n_4\,a = 11.0476$
$n_5\,a = 14.0666$

Since

$$4\,\pi\,q = -\frac{\partial H}{\partial r} \tag{30}$$

and

$$\frac{d J_0\,(n\,r)}{d r} = -\,n \cdot J_1\,(n\,r), \tag{31}$$

$$q = \frac{H_0}{2\,\pi\,a} \sum \frac{e^{-a_p^2 t}\,J_1\,(n\,r)}{(1 + s^2\,n^2)\,J_1\,(n\,a)}. \tag{32}$$

The function defined by (27) satisfies (23) when $r = a$ for all values of t, and equation (6) for all points within the core for all positive values

[11] Byerly: Treatise on Fourier's Series, etc., p. 229.

of t; when $t = 0$, the function is in the core everywhere equal to H_0, and when t is infinite, it vanishes. It is easy to see, therefore, that a function H defined by the equation

$$H \equiv H_\infty - \frac{2\,H_\infty}{a} \sum \frac{e^{-a_p^2 t} \cdot J_0(n_p r)}{n_p (1 + s^2 n_p^2) \cdot J_1(n_p a)}, \tag{33}$$

where H_∞ is a given constant, satisfies equations

$$H_a + s\,[D_r H]_{r=a} = H_\infty$$

and (6), is everywhere equal to zero when $t = 0$, and, when t is infinite, is everywhere equal to H_∞. Since a function which satisfies these conditions is unique, (33) represents the strength of the magnetic field within the core of the solenoid while it is being magnetized from a neutral state to the uniform intensity H_∞ by a current (C) in the coil due to an electromotive force impressed in it. If N is the number of turns of wire in the coil, E the applied electromotive force in the coil circuit, and $w = w' + w''$ the resistance of the coil circuit, all three per centimeter of length of the solenoid, the coil current is given (11) by the equation

$$C = \frac{E}{w} - \frac{\frac{1}{2} N \rho\, a}{w} \left[\frac{2\,H_\infty}{a} \sum \frac{e^{-a_p^2 t}}{1 + s^2 n_p^2} \right]. \tag{34}$$

It is possible to hasten the growth of a current of given final value in a simple circuit with fixed inductance (L) independent of the current strength, by increasing the applied electromotive force (E), and adding to the circuit a corresponding amount of resistance wound non-inductively; for this process decreases the time-constant L/r without changing E/r. The same statement is true in practice for almost every sort of electromagnet.[12] It is not easy to see immediately from equation (34), however, just what the effect on C is of a given change in w, for both n and a involve w implicitly through s.

Since $H_\infty = 4\,\pi\,N\,C_\infty$, we may rewrite (34) in the form

$$C = C_\infty \left[1 - \frac{2\,s}{a} \sum \frac{e^{-a_p^2 t}}{(1 + s^2 n_p^2)} \right]. \tag{35}$$

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[12] After this paper was in type I became acquainted with the results of the elaborate study made by Professor T. Gray into the manner of growth of currents in the coils of electromagnets with finely divided cores. The beautiful curves which he gives in Volume 184 of the Philosophical Transactions of the Royal Society illustrate very strikingly the fact here mentioned.

For $\mu = 40$, and $s = 1$, the series and the parentheses which appear in this equation have at different times the approximate values given in Table VI.

TABLE VI.

t	Σ	$\frac{2s}{a}\Sigma$	$\left[1-\frac{2s}{a}\Sigma\right]$
0.00	6.2500	1.0000	0.0000
0.25	1.2985	0.2078	0.7922
0.50	0.8783	0.1405	0.8595
0.75	0.6743	0.1079	0.8921
1.00	0.5454	0.0873	0.9127
1.25	0.4530	0.0725	0.9275
1.50	0.3814	0.0610	0.9390
2.00	0.2748	0.0440	0.9560
3.00	0.1456	0.0233	0.9767
4.00	0.0775	0.0124	0.9876
6.00	0.0219	0.0035	0.9965
8.00	0.0062	0.0010	0.9990
10.00	0.0017	0.0003	0.9997

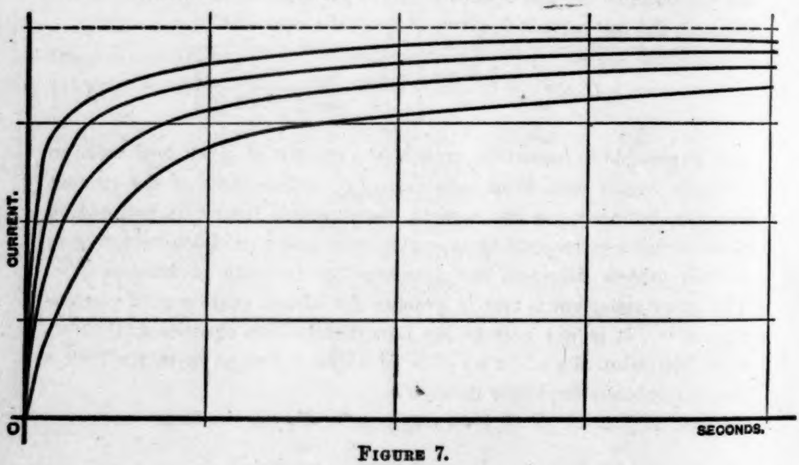

FIGURE 7.

Figure 7 shows C plotted against t for $\mu = 40$, 80, 160, and 320.

Since $4\pi q = -\partial H/\partial r$, it follows from equation (33) that the value of q in the case of the growing C is the negative of the value given by equation (32) for the case of decaying C. Figure 8 shows the value of q at different times for $\mu = 40$, 80, and 160, $r = a/10$, and $s = 1$.

When $s = 2.5$, the equation corresponding to (29) is

$$5 \cdot J_0(z) = z \cdot J_1(z), \tag{36}$$

and the first five roots have approximately the values given below.

TABLE VII.

$n_1 a = 1.9898$
$n_2 a = 4.7132$
$n_3 a = 7.6177$
$n_4 a = 10.6223$
$n_5 a = 13.6786$

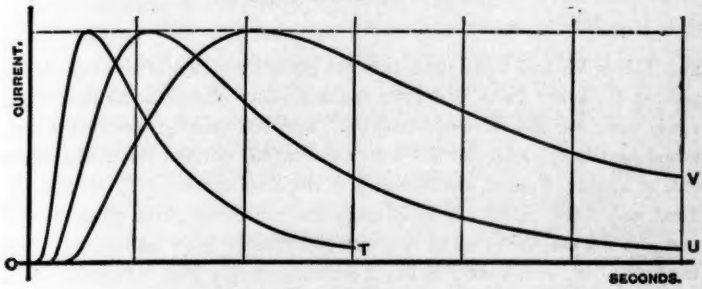

FIGURE 8.

and for $\mu = 40$, the series of (35) has approximately the values given in Table VIII.

TABLE VIII.

t	Σ	$\frac{2s}{a}\Sigma$	$\left[1-\frac{2s}{a}\Sigma\right]$
0.00	2.5000	1.0000	0.0000
0.25	1.0729	0.4292	0.5708
0.50	0.8052	0.3221	0.6779
0.75	0.6542	0.2617	0.7383
1.00	0.5511	0.2204	0.7796
1.25	0.4732	0.1893	0.8107
1.50	0.4111	0.1644	0.8356
2.00	0.3151	0.1260	0.8740
3.00	0.1889	0.0756	0.9244
4.00	0.1137	0.0455	0.9545
5.00	0.0685	0.0274	0.9726
6.00	0.0413	0.0165	0.9835
8.00	0.0150	0.0060	0.9940
10.00	0.0054	0.0022	0.9978

Suppose that in the case of the solenoid the resistance (w') of the coil alone, per centimeter of the length of the core, is such that when there is no outside resistance in the coil circuit, s is 2.5, and let us study the effect upon the manner of growth of C, of adding resistance wound non-inductively to the coil circuit to such an amount (w'') per unit of length of the core that s becomes 1.

From (35), we get

$$\frac{C_\infty - C_t}{C_\infty} = \frac{2s}{a} \sum \frac{e^{-q_p^2 t}}{1 + s^2 n_p^2}, \tag{37}$$

and Tables VI and VIII show that the second member of this equation is greater for every value of t after the beginning when s is 2.5 than when s is 1, whatever the value of μ may be. The first member denotes the fractional part of the final current which the actual current has at the time t still to attain; if, then, the intensity of the final current ($C_\infty = E/w$) be fixed, and if the current be built up in the coil circuit, first, when $w = w'$ and $s = 2.5$ and the value of E is correspondingly low; and second, when $w = w' + w'$, $s = 1$, and E has a correspondingly high value, the actual current will lag behind the final current by a smaller amount at every instant in the second case than in the first. Again, if E be fixed and if different values be given to w so that C_∞ has different values the actual current lags behind the final current by a smaller fraction of the latter at every instant when s is 1 than when $s = 2.5$, that is when w is large than when w is small.

The quantity s, when the geometrical conditions are fixed, is inversely proportional to w, but is independent of μ, as is also n; a^2 is, however, inversely proportional to μ, and for two different values of μ, $e^{-a^2 t}$ would have the same numerical value at times which are to each other as these values of μ. It is sufficiently well proved that the effective permeability of the iron core of an electromagnet, when a current is rising rapidly in the coil, is not always the same as the permeability belonging to the instantaneous value of the current as determined from a statical hysteresis diagram. If in any case where w is fixed the effective value of μ should be greater or smaller for an increase in the value of the applied voltage E, the growth of the current would be relatively retarded or accelerated. We shall find it well to return to this subject later on.

The Effect of Variation in the Permeability of the Core of an Electromagnet upon the Manner of Growth of a Current in the Coil.

We may add to the foregoing theoretical discussion of the effects of Foucault currents in a solid core the permeability of which has a fixed value, a few words upon the manner in which a current might be established in the coil of the magnet described in this paper if there were no eddy currents in the core, as, of course, there really would be in an actual case, unless the core were divided.

After the form of the dotted curve $D\,P$ in Figure 4 had been determined with considerable accuracy (the observations of different days agreeing with each other almost exactly), the curve was plotted on a very large scale by means of a needle point upon thin sheet zinc and an accurate template was then cut out; with the help of this and a metal straightedge, I measured as carefully as I well could the slope (λ) of this curve for a large number of different values of the current (i). Since 2.823×10^9 times an ordinate of the $D\,P$ curve shows the total flux (F) through the coil for a current represented by the corresponding abscissa in amperes, then, if there were no eddy currents in the core and no time-lag in the magnetization of the iron, the building up of a current in the coil under the given circumstances on the application of a steady voltage E in a circuit of total resistance r ohms would be dominated by the equation

$$E - r\,i = 28.23\,\lambda \cdot \frac{d\,i}{d\,t}, \tag{38}$$

(or)

$$\frac{d\,t}{d\,i} = (28.23)\,\frac{\lambda}{E - r\,i}, \tag{39}$$

in which the second member is now a known function of i. I plotted this function and determined by aid of an Amsler's Planimeter the values of t for a number of values of i in the actual case, where E was 84.0 and $r = 13.55$. Figure 9 shows the building-up curve (Q) which this process yields, and also the actual curve[13] carefully reproduced from an oscillograph record.

If we were to define the inductance of the magnet in a condition represented by a point R on the dotted line $D\,P$ in the statical hysteresis diagram, as the ratio of the total flux which then passes through the coils

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[13] J. and B. Hopkinson, The Electrician, September 1892; J. Hopkinson, Wilson and Lydall, Proceedings Royal Society, Vol. 53.

to the intensity of the current belonging to the point, its value would decrease when the current increased, in the manner shown in the subjoined table.

TABLE IX.

Current.	"Inductance" in Henries.	Current.	"Inductance" in Henries.
0.5	169.4	3.5	57.4
1.0	124.2	4.0	51.9
1.5	99.8	4.5	47.5
2.0	84.0	5.0	43.7
2.5	72.3	5.5	40.5
3.0	64.0	6.0	38.1

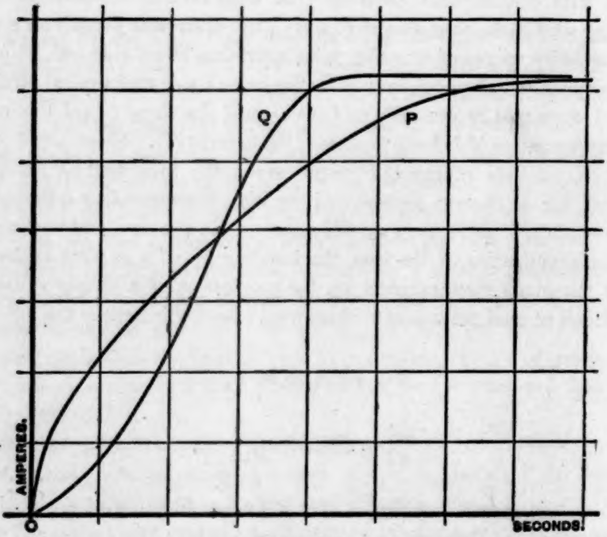

FIGURE 9.

If, however, we write

$$\frac{d(Li)}{dt} = E - ri \tag{40}$$

and plot from the actual building-up curve given in Figure 9, $E - ri$ as a function of t, we may get the area under this curve by means of a planimeter for a series of values of t, and determine L from the equation

$$Li = \int_0^t (E - ri)\, dt. \tag{41}$$

The "apparent inductance" thus defined is affected by Foucault currents; its values for several points of the curve are given in Table X.

TABLE X.

Time.	Current.	L.	Time.	Current.	L.
0.1	0.55	80.2	3.5	4.64	44.1
0.5	1.55	70.4	4.0	5.03	40.9
1.0	2.25	64.0	4.5	5.35	37.9
1.5	2.84	59.1	5.0	5.62	35.1
2.0	3.33	54.9	5.5	5.84	32.4
2.5	3.79	51.9	6.0	5.99	30.0
3.0	4.23	47.4			

L plotted against i yields a curve that is fairly straight.

The theoretical curve (Q) is very like some[14] of the actual curves which have been obtained from magnets with finely divided cores, whereas the actual building-up curve represented in the diagram is similar to those[15] which almost any electromagnet with solid core can be made to yield. The eddy currents, the changes in effective permeability, and the other disturbing influences taken together, do not in such a case cause the curve to deviate very widely in shape from that which one could get from a simple circuit with fixed inductances and no eddy currents: the resemblance can almost always be made close by proper choice of the electromotive force of the exciting battery.

After I had determined hysteresis diagrams (some of which are shown in Figure 3) for the magnet for a large number of gap-widths up to 28 mm., it seemed likely that for an air gap about 35 mm. wide the hysteresis diagram would not be very different from a single straight line. For this gap-width the induction flux through the coil should be practically proportional to the strength of the current and amount to about 6.45×10^6 for a current of one absolute unit so that the inductance of the coil circuit should be about (28.23) (0.645) henries for a wide range of currents. The form of the building-up curve of a current in the coil of an electromagnet generally depends very much upon the magnetic state of the iron at the outset. If a steady current which has been running through the coil for some time be interrupted, and if then after a little the circuit be closed again, the manner of growth of the new current is

[14] See, for instance, the fine diagram given by Dr. Thornton, Phil. Mag., 1904, p. 625.

[15] J. Hopkinson and Wilson, Phil. Trans., 1895, pp. 275, 280.

generally very different if this current has the same direction as its predecessor or the opposite direction: that is, if the magnetism of the core follows the hysteresis diagram in a direction corresponding to DP in Figure 4 or in the direction DQ. An easy way of testing whether the hysteresis curve of a *large* electromagnet has an insignificant area is to obtain large oscillograph records of the building-up curves of direct and reversed currents and to compare the two. I took, therefore, a series of building-up curves for a gap-width of 35 mm., using currents of 2.75 amperes and 5.60 amperes, and found that in both cases the curves were wholly indistinguishable[16] even when enlarged and superposed on the screen, whether the current in question had the same direction as its predecessor or the opposite direction. With this gap-width, therefore, the magnet is an example of a circuit "containing iron" with an induction flux for *steady currents* almost exactly proportional to the strengths of these currents, and in this sense with a fixed inductance. If there were no eddy currents, and no time-lag in the magnetization of the core, the growth of the current in the coil should follow the law

$$C = \frac{E}{r}(1 - e^{-rt/L}),$$

where L is this fixed inductance. Figure 10 shows the actual oscillograph records for 2.75 amperes and 5.60 amperes in full line, and the theoretical curve in dotted line for $E = 80$. In this case, where the "statical effective" value of μ is independent of the current, but where eddy currents and what we may term time-lag in the taking up of the magnetism by the iron may enter, it is interesting to see that in spite of the retardation due to eddy currents, the current in the main circuit builds up more quickly than would correspond to the statical value of μ when the current is 2.75 amperes and that it starts to do so when the current is 5.6 amperes.

The building-up curves shown in this paper are careful reproductions of oscillograph records, of which I have several hundreds. Some of these were obtained with the aid of a Duddell Double Oscillograph, the drum of which could be turned either by an electric motor from the alternating street circuit or by clock-work, but most of them I got with the help of two single instruments made by Mr. J. Coulson, who helped me to take the photographs, and these served their purpose admirably. One of them, which was used in measuring comparatively small induction currents and needed to be very sensitive, was not quite aperiodic when

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[16] For a similar case, see Professor T. Gray, Phil. Trans., Vol. 184.

suddenly deflected to the end of its scale, but this fact did not affect records of the kind used here. This instrument consisted merely of a mirror galvanometer in which the extremely minute magnet and mirror were fastened to a piece of fine stretched gimp damped in oil. I had four drums for carrying the sensitized paper, or the film when this had to be used, and any one of these could be driven very uniformly at almost

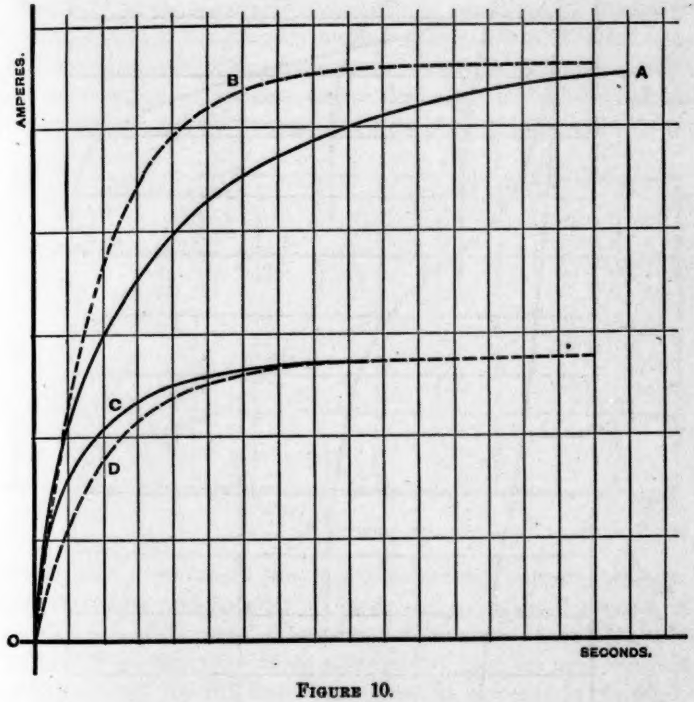

FIGURE 10.

any rate up to a rim velocity of a meter per second by means of chronograph clock-work. Most of the paper used was in strips from 16 to 20 centimeters wide and some of the curves are about a meter long. The deflections of each oscillograph were strictly proportional to the current in its coil. A large zinc template, which carefully kept the irregularities of the record, was made for each curve; this template was used in drawing a large diagram and the figures here given are copies of these diagrams very much reduced. A few of the larger records were redrawn

from measurements made on the photograph itself, but all the smaller ones, and most of the others, were pricked through under a lens by a

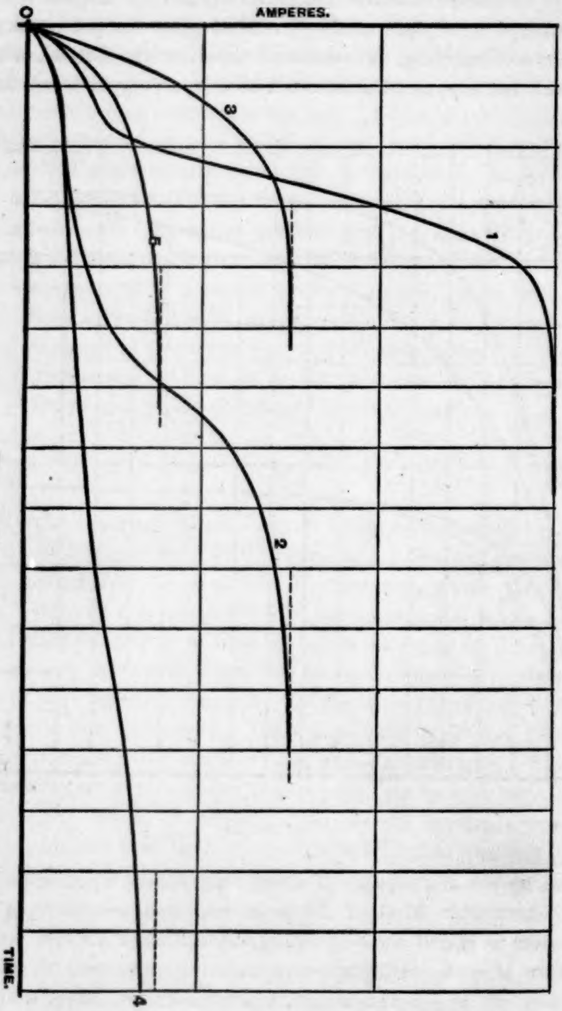

FIGURE 11.

very fine needle point and the record itself was then placed directly in front of the condensing lenses of a large projecting apparatus and thrown

up on paper tacked to a screen; it was generally possible to see the whole of the diagram on the screen besides the bright images of the needle holes, and to reproduce this diagram much enlarged upon the paper. With a given battery, and a given current with a given condition of the iron, it was always easy to get any desired number of records which, when superposed upon the screen, were practically indistinguishable.

Figure 11 shows a series[17] of building-up curves from a 15-kilowatt transformer very kindly placed at my disposal by Mr. S. E. Whiting. The finely laminated core of this transformer has a cross-section area of 108 square centimeters. The same magnetizing coil of about 340 turns was used throughout, but the electromotive force of the storage battery in the

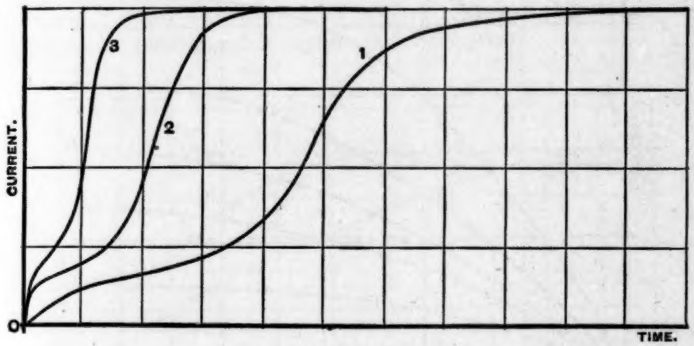

FIGURE 12.

coil circuit could be changed so as to give the current the desired strength. In curve (1) the final intensity of the current was about 3 amperes, in curves (2) and (3) it was 1.5 amperes, and in curves (4) and (5), 0.75 amperes. With each value of the electromotive force the steady current was sent through the coil, first in one direction and then in the other, for a number of times, and was then interrupted for a few moments previous to taking the photographic record from the oscillograph. Curves (1), (2), and (4) were obtained when the current in the coil had a direction opposite to that of the next preceding current; curves (3) and (5), when the current had the same direction as the preceding one. For a current of 0.37 amperes the reversed-current-curve had lost its points of

[17] For a large number of similar curves, see Professor T. Gray's paper in the Philosophical Transactions, Vol. 184.

inflection, and had become everywhere convex upward. With a voltage of only 6 and the low resistance primary of the transformer as exciting coil, the building-up time was an extremely short fraction of a second, and the building-up curve looked like a very straight, and nearly vertical, sign of integration.

If a number of equal coils of wire of a given size, each of resistance r, be wound together uniformly about a wooden ring so as to have equal self-inductances, and if a storage battery of small internal resistance be made to send a current through (say) n of the coils in series, the inductance of the circuit will be nearly proportional to n^2 and the time-constant

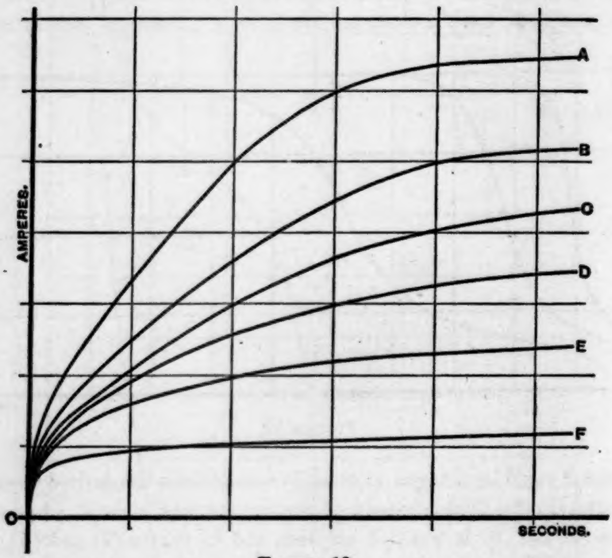

FIGURE 13.

which is independent of the applied electromotive force and therefore of the current, will be nearly proportional to n. If the core of the ring be made of iron, the problem will, of course, be complicated in many ways, but in this connection Figure 12 is interesting; curves 3, 2, and 1 show the manner of growth of a current in coils of about 85, 170, and 340 turns about the core of the transformer just mentioned, in terms of its final value. The electromotive force was in reality the same for all three cases, and the currents were 6 amperes, 3 amperes, and 1.5 amperes.

The Manner of Growth of a Current in the Coil of an Electromagnet which has a Solid Core.

We may now consider some oscillograph records which show the manner in which under given circumstances a current will grow in the coil of the magnet represented by Figure 1. It is evident from what precedes that when other conditions are determined, the magnetic state of the core at the time when the coil circuit is closed will generally influence the result greatly. By sending through the coil a long series of steady currents of gradually decreasing intensity alternately in one direction and the other, one may reduce very low the residual magnetism in the iron even when the gap is closed.

Figure 13 shows building-up curves from a nearly neutral core, under a voltage of about 84, when the air gap was closed, for currents of six different intensities from 1.2 amperes to 6.5 amperes.

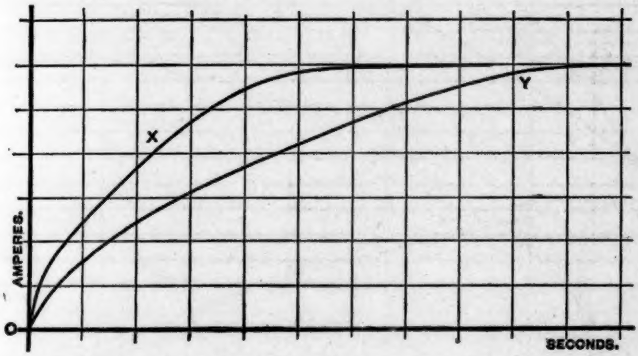

Figure 14.

Figure 14 shows curves obtained with a voltage of about 80 under two different conditions of the core. After the iron had been pretty well demagnetized, the growth of the current when the coil circuit was suddenly closed followed the law indicated by the upper curve: if, when the current had become steady, the circuit was broken and, after fifteen or twenty seconds, closed with the direction of the current reversed, the march of the current followed the lower curve. The width of the gap was 1.6 mm. in this experiment. To study the influence of the width of the air gap upon the shape of the building-up curve of the current in the coil circuit I used first a storage battery of relatively high voltage

(270) and obtained a long series of records for gap-widths up to about 30 mm. When the gap was closed and the current had been sent first in one direction and then in the other alternately for a number of times, I got the results given in Table XI for the growth of a current (*A*) which had the same direction as the next preceding current, and the results given in Table XII for a current (*B*) which had a direction opposite to that of the preceding current.

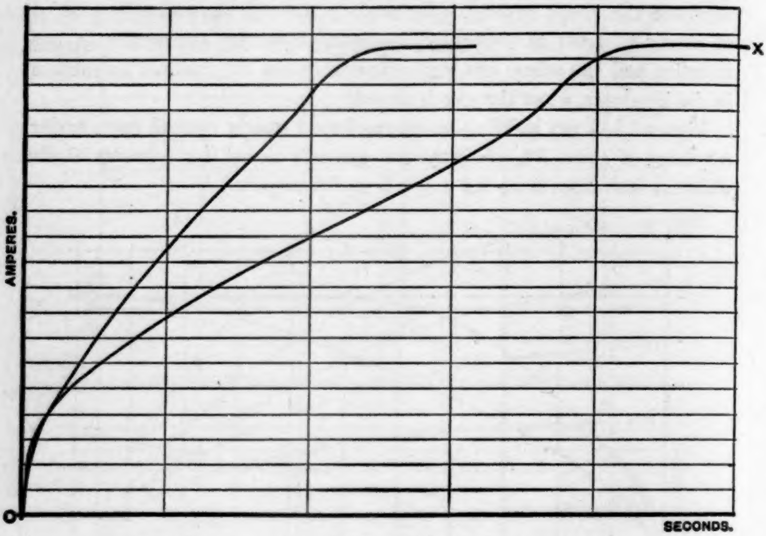

FIGURE 15.

TABLE XI.

A. (DIRECT CURRENT.)

Time in Seconds after the Closing of the Circuit.	Approximate Intensity of the Current in Amperes.	Time in Seconds after the Closing of the Circuit.	Approximate Intensity of the Current in Amperes.
0.10	3.20	1.50	13.95
0.20	4.36	1.75	15.41
0.30	5.37	1.80	15.84
0.40	6.33	1.90	16.42
0.50	7.21	2.10	17.67
0.75	9.07	2.20	18.16
1.00	10.96	2.40	18.56
1.25	12.55	2.50	18.60

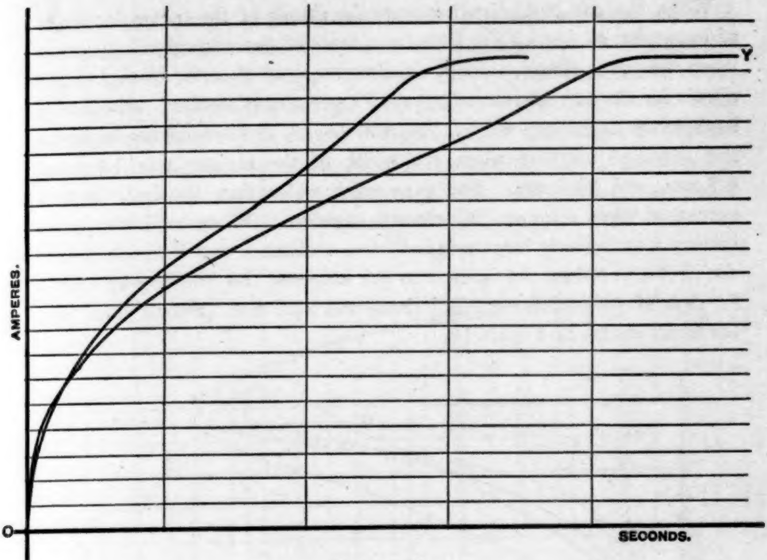

FIGURE 16.

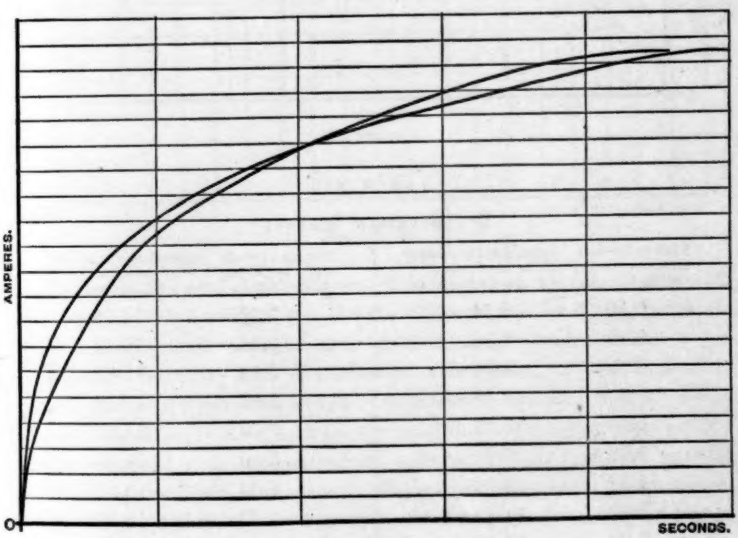

FIGURE 17.

When the gap is relatively narrow the shapes of the curves for direct and reversed currents differ little from those of the corresponding curves when the gap is closed, though the time required after the closing of the circuit for the current to attain (say) 99 per cent of its final value slowly increases with the gap-width. Figures 15, 16, 17 show curves for direct and reversed currents when the width of the air gap was 1.6 mm., 6.6 mm., and 28.3 mm. For gaps much wider than the last, the two curves of each diagram fall closely together. When with the same battery a sufficiently large non-inductive resistance was introduced into the circuit to reduce the current to 8.5 amperes, the building-up curves for gaps of width 1.6 mm. (full lines) and 25.0 mm. (broken lines) had the forms shown in Figure 18.

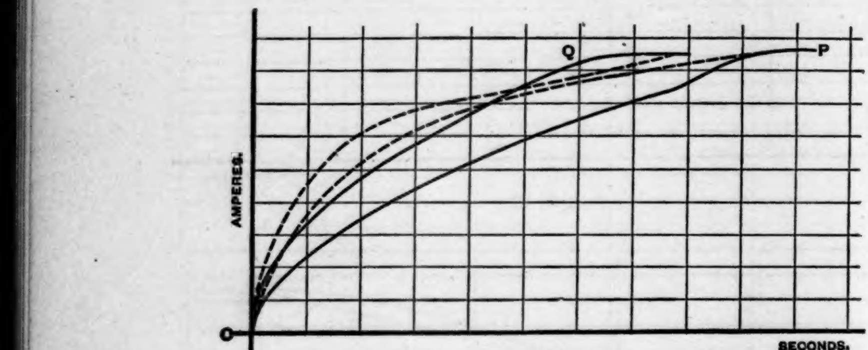

FIGURE 18.

TABLE XII.

B. (REVERSED CURRENT.)

Time in Seconds after the Closing of the Circuit.	Approximate Intensity of the Current in Amperes.	Time in Seconds after the Closing of the Circuit.	Approximate Intensity of the Current in Amperes.
0.10	3.34	3.20	14.76
0.20	4.36	3.40	15.43
0.30	4.94	3.50	15.84
0.40	5.48	3.60	16.48
0.50	5.87	3.70	17.15
1.00	7.99	3.80	17.58
1.50	9.75	4.00	18.31
2.00	11.19	4.30	18.56
3.00	14.12	4.50	18.60

When the voltage of the storage battery was reduced to 90 the building-up curve for a reversed current of 6.9 amperes had the form of the right-hand curve (1) in Figure 19 when the gap was closed, but of the next curve (2) when the air gap was 13 mm. wide. The curves cross eleven seconds after the start. For the current to attain half its final strength, 3.5 seconds are required when the gap is closed, but only 1.5 seconds when the width of the gap is 13 mm.; the current attains 99 per cent of its final value more quickly when the gap is closed than when it is open.

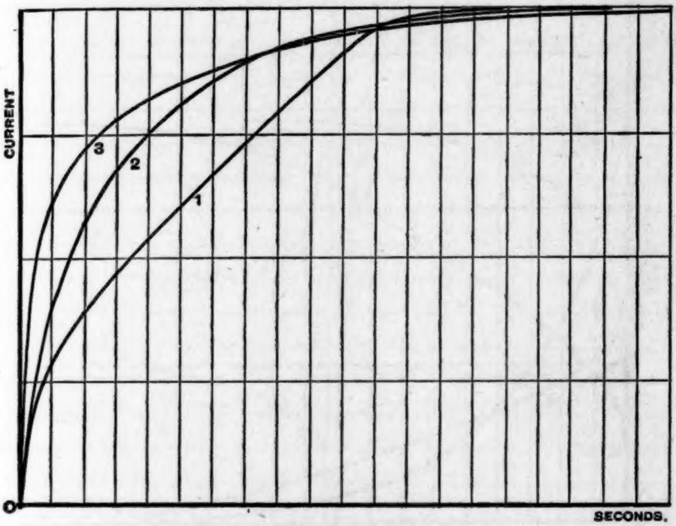

FIGURE 19.

The leftmost curve (3) of Figure 19 belongs to a reversed current of 1.3 amperes when the gap was closed; the ordinates are exaggerated so as to make the final value the same as for the larger current.

When a secondary coil, consisting of a few turns of insulated wire wound around the whole core of the magnet, was connected with an oscillograph which made its record on the same sheet of paper as the oscillograph connected with the main circuit, it was easy to get the rate of growth of the induction flux in the core. Figure 20 shows building-up curves when the gap was closed for a current of 1.3 amperes furnished (A) by a battery of 40 storage cells and (D) by a battery of 10 storage cells; in

the first case it was necessary of course to add non-inductive resistance to the circuit to reduce the current to this value. The time required for the current to attain practically its final value is far less in the first case than in the second. The dotted curves show the records of the oscillograph in the secondary circuit on an arbitrary scale. The two curves are quite unlike in shape, but the areas under them, as measured by a planimeter, are almost exactly the same. The general forms of the dotted curves here shown are like all the scores of others which I have obtained under all sorts of conditions of current strength and resistance.

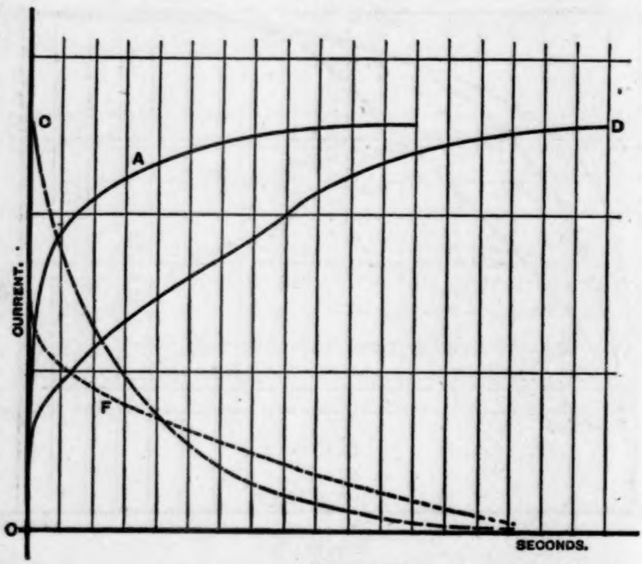

FIGURE 20.

The pole pieces of the magnet shown in Figure 1 are fastened to the cylindrical arms by long bolts which extend outside the frame and carry nuts which press upon the yokes and serve to keep the jaws apart. When one of these bolts was removed it left a long axial hole of about an inch in diameter through the arm, and into this I put a long rod of soft iron upon which a layer of fine insulated wire had been wound pretty uniformly, and this coil was connected with the secondary oscillograph already mentioned. This coil and its long core filled the cavity fairly completely except at the outside ends, but doubtless the joint at the inner

end of the core affected the results somewhat, and in a manner not easy to treat mathematically; nevertheless the indications of this coil were interesting and characteristic in their general features, as we shall see

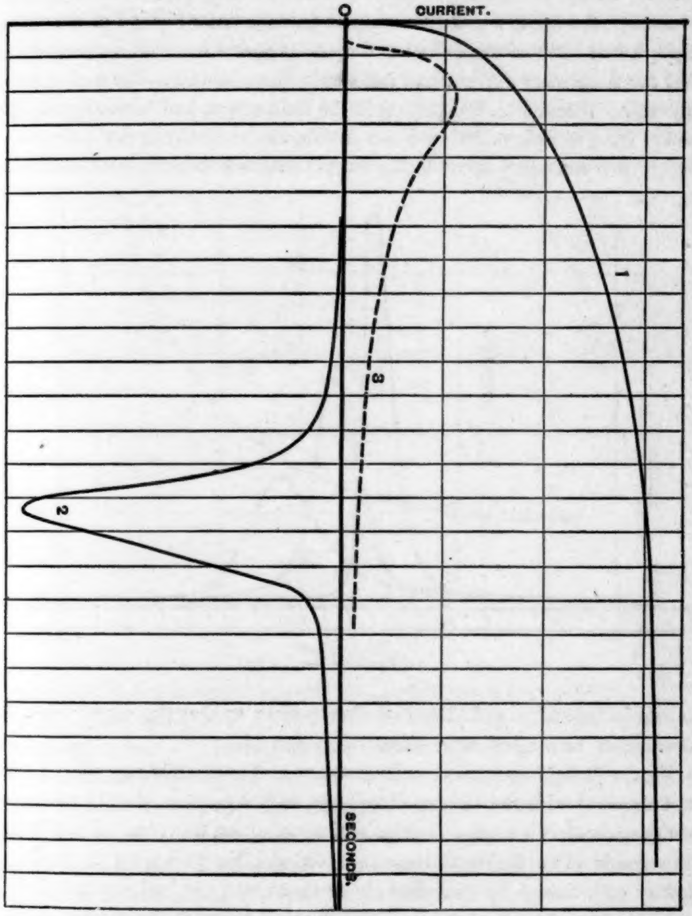

FIGURE 21.

hereafter, of the behavior of the eddy currents in the core near its axis. When a current was started in the coil circuit, the mirror of the oscillograph attached to the secondary coil remained sensibly at rest for an

interval the length of which depended upon the final value of the coil current, the electromotive force in its circuit, and the magnetic condition of the core at the start; then a temporary current which had a sensible value for perhaps ten or fifteen seconds passed through the secondary, though if a sensitive ballistic galvanometer instead of the oscillograph had been connected with the coil it would have been made clear that the temporary current had not wholly disappeared at the end of this interval. When, after the current in the main circuit had been apparently steady for a minute or two the coil circuit was suddenly broken, the current in the secondary became almost immediately evident, soon attained

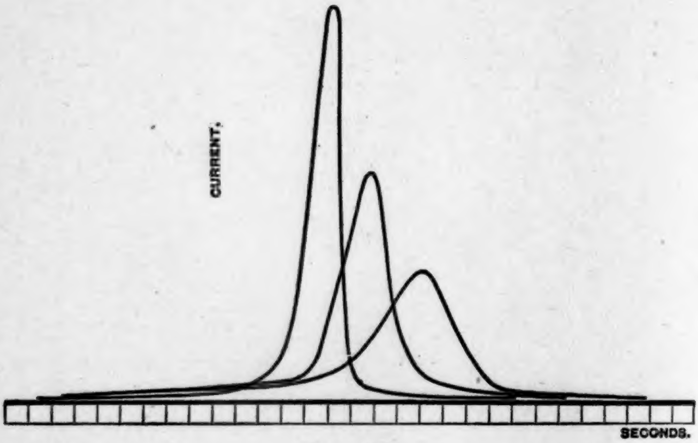

FIGURE 22.

its maximum value, and then died slowly away so that the mirror seemed to reach its zero again after about thirty seconds.

Figure 21 shows a typical oscillograph record accurately reproduced. It was obtained with the air gap closed and with a current of 3.12 amperes in the main circuit from a storage battery of about 84 volts. For five or six seconds after the start there was no sensible indication in the secondary coil, though by that time the main current (1) had attained about three fourths of its final value. The greatest value of the current in the secondary did not occur until rather more than fourteen seconds after the current in the primary had begun, and the secondary current had apparently died out in less than thirty seconds from the beginning. The ordinates of the curve of the current in the main circuit represent amperes;

the ordinates of the curve (2) which represents the current in the secondary, are on an arbitrary scale. If at the time $t = 0$, the main circuit through which a steady current of 3.12 amperes had been passing were suddenly broken, the current in the secondary would have grown and died out after the manner indicated by the dotted curve (3). For about half a second after the main circuit was interrupted, there would have been no sensible current in the secondary, but then it would have suddenly appeared in a manner that strongly suggests the theoretical curves shown in Figures 6 and 8. Figure 22 illustrates the fact that the form of the secondary current curve depends very much upon the intensity of

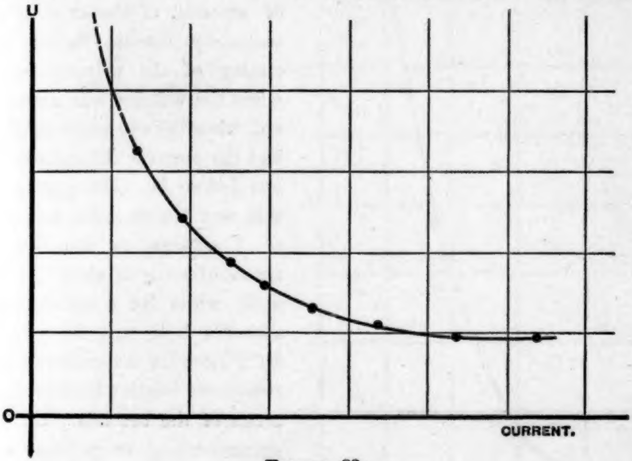

FIGURE 23.

the current in the main circuit, and that sometimes a comparatively slight change in the latter will alter materially the maximum intensity of the secondary current. The diagram is a careful reproduction of the records of the secondary oscillograph for currents of 3.76 amperes, 3.12 amperes, and 2.60 amperes respectively. The scale at the bottom represents seconds; the records, which are to be read from right to left, are displaced with respect to each other merely to prevent confusion in the figure.

The lag in seconds of the crest of the current in the secondary circuit behind the closing of the main circuit depends, naturally enough, upon the final intensity of the main current and upon the magnetic condition of the core at the outset. If a series of currents of the same final intensity be passed first in one direction and then in the other through the main coil, and if after the circuit has been broken for half a minute the

current be sent through the coil again, the retardation of the crest of the secondary will be perhaps twice or thrice as great if the direction of the new current be opposed to that of the next preceding one, as it would be if the new current had the same direction as its predecessor.

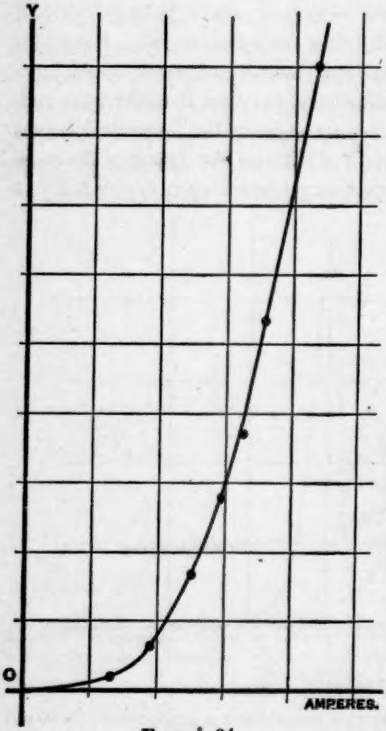

FIGURE 24.

In Figure 23 the abscissas represent currents in amperes and the ordinates the lag, in tens of seconds, of the crest of the secondary current behind the closing of the primary circuit when the voltage was about 84 and when the current considered had the opposite direction to the one before it. The gap was in this case closed. For a current of 1 ampere, as the diagram shows, the lag is about 40 seconds, while for a current of 5 amperes it is only 10 seconds. In Figure 24 the ordinates represent the relative heights of the crests of the secondary currents corresponding to primary currents the intensities of which, in amperes, are represented by the abscissas.

In the next portion of this paper I hope to discuss some of the results given here, with a large number of others which throw some light upon the march of the eddy currents in a massive iron core of the kind here used. In this connection the records of the secondary coils wound upon the separate members of a set of loosely-fitting, coaxial, cylindrical shells of soft steel, made part of the core of the magnet represented in Figure 1, will be interesting.

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